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A CALCULATION OF THE PARAMETERS OF THE HIGH-SPEED JET FORMED IN THE COLLAPSE OF A BUBBLE

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UDC 532.529.6

As is known, the collapse of vapor bubbles in a liquid can cause the intensive destruction of solid boundary surfaces. Experimental and theoretical investigations of bubble collapse have led to the conclusion that the surface of a bubble can deform and a liquid jet directed toward the solid surface can form in the process [1, 2]. In the theoretical reports [3, 4] too low jet velocities were obtained, inadequate to explain the destruction of the surface in a single impact. In [5] it was found as a result of numerical calculations that the formation of jets possessing enormous velocities is possible. It was also found that two fundamentally different schemes of jet formation are possible in the collapse of a bubble near a wall. The transition from one scheme to the other occurs upon a relatively small change in the initial shape of the bubble. In the present report we investigate the case of sufficiently small initial deformations of a bubble when the region occupied by the bubble remains simply connected during the formation of the jet; i.e., the separation of a small bubble from the bubble does not occur. In the case of the second scheme of bubble collapse near a wall the connectedness of the free boundary is disrupted and a small bubble separates off during the formation of the jet.

In an ideal incompressible liquid, bounded by a plane solid surface and stationary at infinity, there is a bubble. At the boundary of the bubble the liquid pressure is $p=0$ and at infinity $p=p_\infty$. At the starting time $t=0$ the shape and position of the bubble are given. It is required to determine the motion of the liquid and the shape of the bubble boundary S at $t>0$.

The motion of the liquid was calculated numerically on a BÉSM-6 computer using the method of calculating the potential motions of a liquid with free boundaries suggested in [6]. In the axisymmetric problem the bubble contour is represented with the help of interpolation on a large number of reference points (from

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17 to 41). The unknown quantities are the coordinates of the reference points and the values of the velocity potential at these points. The nonsteady problem was solved by a specially developed explicit scheme. At each step in time an integral equation of the first kind was solved for the relative normal velocity of the liquid at the contour at the values of the potential known at a fixed time.

The calculations were made in the dimensionless variables of length x and time t, connected in the following way with the corresponding dimensional variables:

$$x' = xa,\ t' = ta\sqrt{\rho/p_\infty},\quad v' = v\sqrt{p_\infty/\rho},$$

where v is the dimensionless velocity; a is the size of the bubble.

Smooth initial deformation of the bubble was allowed for. This deformation can develop owing to the presence of pressure gradients in a stream, owing to asymmetry of the flow near the bubble, growing near the wall, and other factors. It is assumed that the bubble has the shape of an ellipsoid of rotation with semiaxes equal to a and b. The semiaxis a is perpendicular to the wall. The solution of the problem depends on the ratio $\chi = b/a$ of the semiaxes and on the distance z_0 of the center of the ellipsoid from the plane.

Calculations of the process of bubble collapse were made in a wide range of variation of the distance z_0 from the plane. The solution of the problem of the collapse of a spherical bubble ($\chi = 1$) with $z_0 = 1.025$, when the bubble is practically in contact with the wall, is presented in Fig. 1. The wall corresponds to the horizontal line. The positions 1-11 of the bubble boundary are shown at the successive times t=0, 0.76, 0.935, 0.995, 1.015, 1.02906, 1.04281, 1.05406, 1.06656, 1.07656, and 1.08656. It is seen that a jet moving toward the solid boundary is formed in the process of collapse. The velocity of the tip of the jet (later called simply the jet velocity) grows sharply as the cavity boundary passes from position 3 to position 5 and then the jet velocity remains practically constant. Position 5 corresponds to the appearance of the initial section of the jet, when the pressing in of the surface is very small. A very small separation of the accelerating mass of liquid from the surface of the bubble is sufficient for its velocity to cease to grow.

The results presented in Fig. 1 agree with the results of [4]. The difference consists in the absence of an irregular surge at the tip of the jet. The problem of the collapse of a spherical bubble near a wall was also solved in [7]. The results of this report differ very strongly from those presented above and from the results of [4, 5]. The formation of the jet occurs at the moment when the bubble has twice as great a size, and the diameter of the jet proves to be considerably smaller. Smoothing in a calculation over some strongly unstable difference scheme is used in [7]. This may be one of the reasons for the disagreement of the results; another reason evidently is the strong dependence of the results on the initial shape of the bubble and the resulting increased demands on the accuracy of the calculations.

With an increase in the initial distance z_0 of a spherical bubble from the plane the minimum bubble size at which the formation of a jet occurs decreases while the jet velocity v increases (Fig. 2).

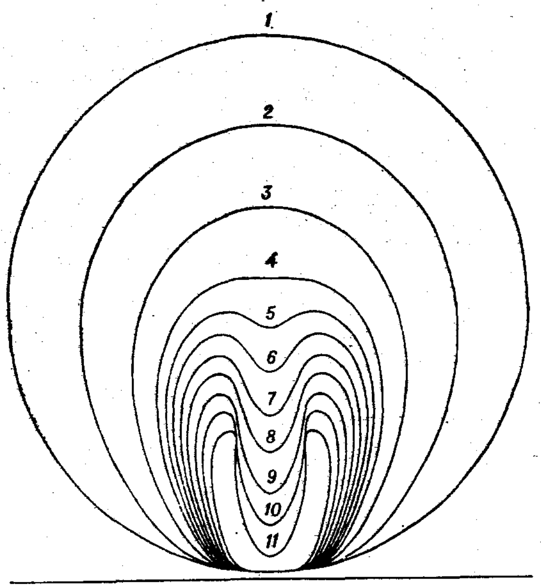

Fig. 1

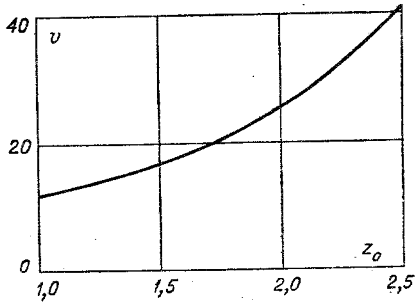

Fig. 2

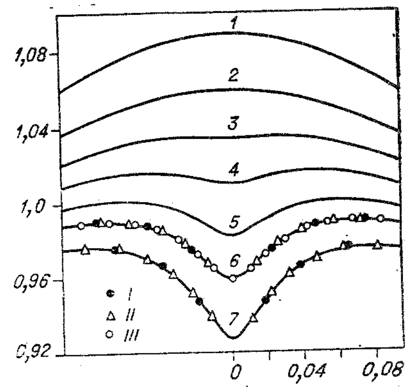

Fig. 3

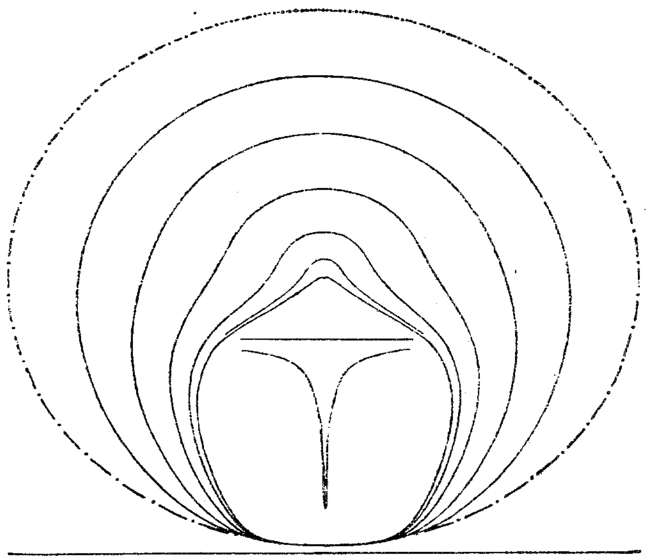

Fig. 4

The allowance for the initial ($t=0$) deformation of the cavity is of the greatest interest, as follows from [5]. Even a small deformation essentially alters the process of jet formation, which is shown in Fig. 3 in the case of a degree of deformation $\chi=1.1$. The distance from the plane is plotted vertically and the distance from the axis of symmetry is plotted horizontally. The positions 1-7 of the bubble boundary correspond to the times $t=1.08344$, 1.08578, 1.08734, 1.08859, 1.08984, 1.09047, and 1.09203. The jet velocity grows about two-fold as the free boundary passes from position 1 to position 4 and then it remains practically constant.

The calculations were made with different degrees of bunching of the reference points in the region of the contour where the appearance of a jet is likely (see Fig. 3). Variant I corresponds to a distance $x_2=0.006047$ between the point nearest the pole and the axis of symmetry at $t=0$; variant II corresponds to $x_2=0.00346$; variant III corresponds to $x_2=0.0025978$. It is seen from the data of Fig. 3 that the results of calculations with considerably different arrangements of the reference points along the length of the contour coincide. This additionally confirms the correctness of the calculations. A control of the accuracy of the calculations was also made at each step by testing the conservation of energy of the system.

In the case of an initial deformation $\chi=1.175$ the nonlinear process of bubble collapse gives an almost needle-shaped jet. The process of bubble collapse and the velocity profile at the free boundary at the first moment after the separation of the initial part of the jet from it at $t=1.14702$ are represented in Fig. 4. The velocity maximum at the center is very sharply expressed. The calculations are greatly hindered by the small scale of the jet which forms.

The characteristic thickness d of the jet depends strongly on the initial degree of deformation χ (Fig. 5). The quantity d is defined as the width of the jet at half its length at the moment when the amplitude of the pressing in of the surface of the bubble vertically (the length of the jet) comprises a value on the order of d. Evidently, the thickness d of the jet is reduced to zero somewhere in the region of $\chi \geqslant 1.175$.

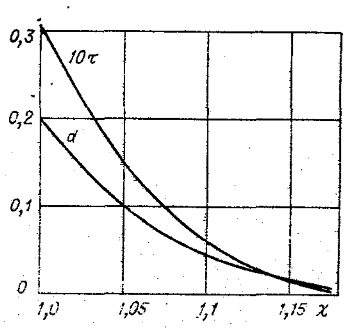

Fig. 5

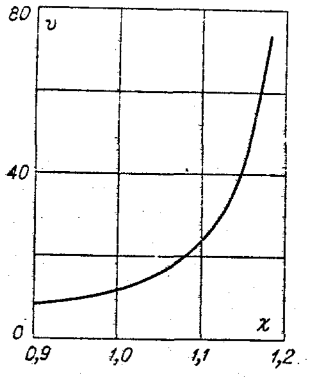

Fig. 6

The time in which the main growth of the jet velocity occurs is an important characteristic of the process of generation of the jet. We define the characteristic time of generation of a jet as the time in which the jet velocity grows from 0.45v to 0.9v, where v is the maximum value of the velocity. The generation time τ depends strongly on the degree of deformation χ of the bubble (see Fig. 5). It is seen that as χ approaches a value of $\chi \approx 1.2$ the generation time decreases sharply and proves to be an order of magnitude less than when $\chi = 1$.

The dependence of the jet velocity on the degree of deformation (Fig. 6) is of the grestest interest. For a prolate ellipsoid of rotation ($\chi < 1$) the values of the jet velocity are smaller than in the case of a sphere, and for an oblate ellipsoid they are larger.

The increase in velocity with an increase in χ is so intense that a value of $v = 11.9$ with $\chi = 1$ (spherical bubble) proves to be completely episodic, not characteristic of the problem of the collapse of a slightly deformed bubble. With the approach to $\chi \approx 1.2$ the jet velocity grows sharply and evidently goes to infinity. For values of $v \approx 100$, corresponding to velocities of about 1 km/sec for water at $p_\infty = 1$ atm and $\rho = 1$ g/cm^3, the compressibility of the water must be taken into account. In the impact of a water jet moving with a velocity of such an order against a solid surface an impact pressure of up to several tens of thousands of atmospheres may be reached.

From qualitative considerations it can be concluded that in the generation of a jet the parameter $\tau v/d$ should remain about the same in different variants. Actually, the quantity $\tau v/d$ remains within limits of 2-4 when the velocity varies almost 10-fold and the generation time of the jet varies 100-fold.

A question can arise: Does surface tension affect the process of generation of a jet? Estimates show that the surface tension σ is important if $v^2 d \lesssim \sigma/(a p_\infty)$ where v and d are the dimensionless velocity and diameter of the jet; a is the size of the bubble. The quantity $v^2 d$ takes values of 29 when $\chi = 1.28$ and 33 when $\chi = 1.15$ and 1.175, respectively; i.e., $v^2 d$ remains almost constant with an increase in χ. Consequently, if for a given bubble size the surface tension was not important in the case of the formation of a broad jet during the collapse of a spherical bubble, then it also will not be important in the case of the formation of an anomalously thin jet because of its high velocities (high pressures develop in the small region where the jet is formed).

The results presented confirm the conclusion of [5] that there is a critical value of the initial degree of deformation $\chi = \chi_* \approx 1.2$ upon approaching which the velocity of the forming jet grows and its thickness decreases. The jet velocity goes to infinity at $\chi = \chi_*$ and the point $\chi = \chi_*$ is singular.

The collapse of a bubble with a deformation greater than critical takes place by the scheme found in [5]. A small jet is formed as a secondary jet as a result of the collapse of an annular jet, cutting off a small bubble from the bubble.

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ASYMPTOTIC THEORY OF THE FLOW AROUND AN OBSTACLE BY A SONIC FLOW

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The first investigation of the problem of the flow around an obstacle by a gas flow whose velocity is equal to the speed of sound at infinity was carried out in [1, 2], where it is shown in particular that the principal term of the appropriate asymptotic expansion is a self-similar solution of Tricomi's equation, to which the problem reduces in the first approximation upon a hodographic investigation. The requirement that the stream function be analytic as a function of the hodographic variables on the limiting characteristic was an important condition determining the selection of the self-similarity exponent n (xy^{-n} is an invariant of the self-similar solution). The analytic nature of the velocity field everywhere in the flow above the shock waves, which arise from necessity upon flow around an obstacle, follows from this condition. The latter was found in [3], where one of the branches of the solution obtained in [1] was used in the region behind the shock waves. The principal and subsequent terms of the asymptotic expansion describing a sonic flow far from an obstacle were discussed in [4], where the author restricted himself to Tricomi's equation. Each term of the series constructed in [4] contains an arbitrary coefficient (we will call it a shape parameter) which is not determined within the framework of a local investigation, and consideration of the problem of flow around a given obstacle as a whole is necessary in order to determine these shape parameters. It follows from the results of [4] that the problem of higher approximations to the solution of [1] coincides with the problem of constructing a flow in the neighborhood of the center of a Laval nozzle with an analytic velocity distribution along the longitudinal axis (a Meyer-type flow). Along with the Meyer-type flow in the vicinity of the nozzle center, which corresponds to a self-similarity exponent n = 2, two other types of flow are asymptotically possible with n=3 and 11, given in [5]. The appropriate solutions are written out in algebraic functions in [6]. The results of [5] show that the condition that the velocity vector be analytic on the limiting characteristic in the flow plane is broader than the condition that the stream function be analytic as a function of the hodographic variables, which is employed in [1, 2, 4]. Therefore, the necessity has arisen of reconsidering the problem of higher approximations for the obstacle solution of F. I. Frankl'. It has proved possible for the region in front of the shock waves to use a series which is more general than in [4], which implies the inclusion of an additional set of shape parameters. The solution is given in the hodograph plane in the form of the sum of two terms; the series discussed in [4] corresponds to the first one, and the series generated by the self-similar solution with n = 3 or with n = 11 corresponds to the second one.

1. Two dimensional irrotational flows of an ideal perfect gas are described in the transonic approximation by the equations [7]

$$-uu_x + v_y = 0,\ u_y - v_x = 0, \tag{1.1}$$

where x and y are the reduced Cartesian coordinates and u and v are the dimensionless components of perturbations of a uniform sonic flow.

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